

IV. CONSTRUCTION OF (e-p-v) EQUATION OF STATE

The limited number of experimental Hugoniot points and the restricted range of data along the atmospheric isobar prohibit the construction of an equation of state solely from experimental data. It is important, however, to use both shock wave and static data to indicate the most appropriate form of the (e-p-v) equation of state. A graphic fit of Hugoniot data in the 200-kbar regime, without the three points from shots 12,326 and 12,496 that indicate crossing Hugoniot curves, suggests a linear dependence of internal energy on pressure along lines of constant volume-isochores; static data show that the partial derivative $(\partial e/\partial p)_v$ varies along the atmospheric isobar. Thus, the (e-p-v) data were fitted to the form

$$e = pf(v) + g(v) \quad (11)$$

with $(\partial e/\partial p)_v = f(v) > 0$ everywhere in the region of interest.

Additional properties of this model follow from thermodynamic relationships. The relationship between specific heat at constant pressure C_p and specific heat at constant volume C_v is

$$C_p = C_v [1 + T(\partial v/\partial T)_p / f(v)] \quad (12)$$

and C_v is constant along an isentrope, since

$$\left(\frac{\partial C_v}{\partial T}\right)_s = \left(\frac{\partial^2 e}{\partial p^2}\right)_v \left(\frac{\partial p}{\partial T}\right)_v^2 = 0 \quad (13)$$

The equation for a Hugoniot curve centered at $(p_0 = 0, v_0)$ is

$$p[f(v) - \frac{1}{2}(v_0 - v)] = g(v_0) - g(v) \quad (14)$$

the differential equation for an isentrope is

$$\left(\frac{\partial p}{\partial v}\right)_s = -\frac{p(1 + df/dv) + dg/dv}{f(v)} \quad (15)$$

and the equation⁹ obtained by formal integration of Eq. 15 shows that the first derivative of $g(v)$ must be positive, i.e., $dg/dv > 0$. The rapid increase of pressure along an isentrope indicates that the (e-p-v) relationship will satisfy the mechanical stability condition $(\partial p/\partial v)_s < 0$ if $f(v)$ satisfies the condition $(1 + df/dv) > 0$.